Let $c_1$ and $c_2$ be real numbers such that $x^3+c_2x^2+c_1x-1=0$ has a real root $0<\xi<1$. Consider the following recursions

$$P_i(x) = -c_2P_{i-1}(x) - c_1P_{i-2}(x) + P_{i-3}(x), \quad i=1,2,\dots,$$

$$Q_i(x) = c_1Q_{i-1}(x) + c_2Q_{i-2}(x) + Q_{i-3}(x), \quad i=1,2,\dots,$$

$$R_i(x) = c_1R_{i-1}(x) + c_2R_{i-2}(x) + R_{i-3}(x), \quad i=1,2,\dots,$$

where

$$P_0(x)=x^2,\ P_{-1}(x)=x,\ P_{-2}(x)=1,$$

$$Q_0(x)=c_1x-1,\ Q_{-1}(x)=x,\ Q_{-2}(x)=0,$$

$$R_0(x)=c_1x^2,\ R_{-1}(x)=x^2,\ R_{-2}(x)=-1.$$

Put

$$K_{i,p}=|P_i(\xi)|\|P_i\|_p^2, \quad K^*_{i,p}=\||Q_i(\xi)||q_i|^{1/2}, |R_i(\xi)||q_i|^{1/2}\|_p,$$

where $\|\cdot\|_p$ is the $\ell^p$- norm and $q_i$ is the leading coefficient of $Q_i$.

Statement 1. *Let $c_1$ and $c_2$ be nonnegative integers such that*

$$c_1 \geq \begin{cases} 1 & \text{if} \quad c_2=0 \\ \lfloor c_2^2/4 \rfloor & \text{if} \quad 1\leq c_2\leq 8 \\ \lfloor c_2^2/4 \rfloor + 1 & \text{if} \quad c_2>8. \end{cases} \tag{1}$$

*Then*

$$\sup K_{i,2}<\infty \quad \text{and} \quad \sup K^*_{i,2}<\infty.$$

*Moreover,*

$$\sup K_{i,2}<1 \quad \text{and} \quad \sup K^*_{i,2}<1$$

*if*

$$c_1 \geq \begin{cases} \lfloor c_2^2/4 \rfloor + 1 & \text{if} \quad 0\leq c_2\leq 8 \\ \lfloor c_2^2/4 \rfloor + 2 & \text{if} \quad c_2>8. \end{cases} \tag{2}$$

Suppose $c_1$ and $c_2$ satisfy (1) or (2). Put

$$I=\inf K_{i,2}, \quad S=\sup K_{i,2}, \quad \tilde{K}_{i,2}=(K_{i,2}-I)/S,$$
$$I^*=\inf K^*_{i,2}, \quad S^*=\sup K^*_{i,2}, \quad \tilde{K}^*_{i,2}=(K^*_{i,2}-I^*)/S^*$$

and create histograms for $\{\tilde{K}_{i,2}\}$ and $\{\tilde{K}^*_{i,2}\}$.

Statement 2. *A frequency curve for $\{\tilde{K}_{i,2}\}$ and $\{\tilde{K}^*_{i,2}\}$ is* $f(x)=\dfrac{1}{\pi\sqrt{x(1-x)}}$.

Statement 3. *Let $c_1=1, c_2=0$ or $c_1=0, c_2=1$. Then there is a subsequence $\tilde{P}_i$ of $P_i$ such that*

*(i)* $|\tilde{P}_1(\xi)|>|\tilde{P}_2(\xi)|>\ldots>|\tilde{P}_i(\xi)|>\ldots,$

*(ii)* $\|\tilde{P}_1\|_\infty<\|\tilde{P}_2\|_\infty<\ldots<\|\tilde{P}_i\|_\infty<\ldots,$

*(iii) for any nonzero $\tilde{P}\in\mathbb{Z}[x]$ with $\|\tilde{P}\|_\infty<\|\tilde{P}_{i+1}\|_\infty$ we have $|\tilde{P}(\xi)|\geq|\tilde{P}_i(\xi)|$.*