

Teachers' mistakes as a teaching tool

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We have:

We have:

$$\lim_{x \rightarrow -\infty} (\sqrt{x^2 + x} - x)$$

We have:

$$\lim_{x \rightarrow -\infty} (\sqrt{x^2 + x} - x) = \lim_{x \rightarrow -\infty} \frac{(\sqrt{x^2 + x} - x)(\sqrt{x^2 + x} + x)}{\sqrt{x^2 + x} + x}$$

We have:

$$\begin{aligned}\lim_{x \rightarrow -\infty} (\sqrt{x^2 + x} - x) &= \lim_{x \rightarrow -\infty} \frac{(\sqrt{x^2 + x} - x)(\sqrt{x^2 + x} + x)}{\sqrt{x^2 + x} + x} \\ &= \lim_{x \rightarrow -\infty} \frac{x^2 + x - x^2}{\sqrt{x^2 + x} + x}\end{aligned}$$

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Calculus I

We have:

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$$f(x) = \sqrt{x^2 + x} - x = \lim_{x \rightarrow -\infty} \frac{x^2 + x - x^2}{\sqrt{x^2 + x} + x}$$

$$f(-100) = \lim_{x \rightarrow -\infty} \frac{x}{\sqrt{x^2 + x} + x}$$

$$= \lim_{x \rightarrow -\infty} \frac{x}{x \sqrt{1 + 1/x} + x}$$

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$$f(-100) \approx 199.5 = \lim_{x \rightarrow -\infty} \frac{x}{\sqrt{x^2 + x} + x}$$

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THEOREM: If $b > 0$, $b \neq 1$, $a > 0$, $c > 0$, and r is any real number, then:

- $\log_b(ac) = \log_b(a) + \log_b(c)$
- $\log_b(a/c) = \log_b(a) - \log_b(c)$
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Calculus I

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$$f(x \pm y) = f(x) \pm f(y)$$

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$f(x \pm y) \stackrel{?}{=} f(x) \pm f(y)$

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$\sqrt{x \pm y} \neq \sqrt{x} \pm \sqrt{y}$

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$(x \pm y)^2 \neq x^2 \pm y^2$

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$\sin(x \pm y) \neq \sin(x) \pm \sin(y)$

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$\cos(x \pm y) \neq \cos(x) \pm \cos(y)$

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■ $\log_b(1/c) = -\log_b(c)$

$\tan(x \pm y) \neq \tan(x) \pm \tan(y)$

We have:

We have:

$$\int \sin 2x dx$$

We have:

$$\int \sin 2x dx = \int 2 \sin x \cos x dx$$

We have:

$$\begin{aligned}\int \sin 2x dx &= \int 2 \sin x \cos x dx \\ &= \left[\sin x = u \right]\end{aligned}$$

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$$= \int 2u du$$

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We have:

$$\int \sin 2x dx = \int 2 \sin x \cos x dx$$

$$= \left[\begin{array}{l} \sin x = u \\ d \sin x = du \\ \cos x dx = du \end{array} \right]$$

$$= \int 2u du$$

$$= u^2 + C$$

Calculus I

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We have:

$$\begin{aligned}\int \sin 2x dx &= \int 2 \sin x \cos x dx \\ &= \left[\begin{array}{l} \sin x = u \\ d \sin x = du \\ \cos x dx = du \end{array} \right] \\ &= \int 2u du \\ &= u^2 + C \\ &= \sin^2 x + C\end{aligned}$$

Calculus I

We have:

$$\int \sin 2x dx$$

$$\int \sin 2x dx = \int 2 \sin x \cos x dx$$

$$= \left[\begin{array}{l} \sin x = u \\ d \sin x = du \\ \cos x dx = du \end{array} \right]$$

$$= \int 2u du$$

$$= u^2 + C$$

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Calculus I

We have:

$$\int \sin 2x dx = \frac{1}{2} \int 2 \sin 2x dx$$

$$\int \sin 2x dx = \int 2 \sin x \cos x dx$$

$$= \left[\begin{array}{l} \sin x = u \\ d \sin x = du \\ \cos x dx = du \end{array} \right]$$

$$= \int 2u du$$

$$= u^2 + C$$

$$= \sin^2 x + C$$

Calculus I

We have:

$$\int \sin 2x dx = \frac{1}{2} \int 2 \sin 2x dx$$

$$= \left[2x = u \right]$$

$$\int \sin 2x dx = \int 2 \sin x \cos x dx$$

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We have:

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$$= \begin{bmatrix} 2x = u \\ d2x = du \\ 2dx = du \end{bmatrix}$$

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$$= \begin{bmatrix} 2x = u \\ d2x = du \\ 2dx = du \end{bmatrix}$$

$$= \frac{1}{2} \int \sin u du$$

$$= -\frac{1}{2} \cos u + C$$

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1. There is a mistake.

Calculus I

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3. $-\frac{1}{2} \cos 2x + \frac{1}{2} = \sin^2 x$.

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$$\begin{aligned} -1 &= (-1)^1 = (-1)^{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} \\ &= (-1)^{\frac{1}{6} \cdot 2 + \frac{1}{6} \cdot 2 + \frac{1}{6} \cdot 2} \end{aligned}$$

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We have:

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PROOF (ELEMENTARY, SHORT): Put

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$$f(a_1, a_2, \dots, a_{n-1}, a_n) > f\left(\frac{a_1 + a_n}{2}, a_2, \dots, a_{n-1}, \frac{a_1 + a_n}{2}\right) \quad \blacksquare$$

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THEOREM (Extreme-Value Theorem): If $f(x_1, \dots, x_n)$ is continuous on a closed and bounded set R , then f has both an absolute maximum and an absolute minimum on R .

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PROOF (ELEMENTARY, SHORT): Put

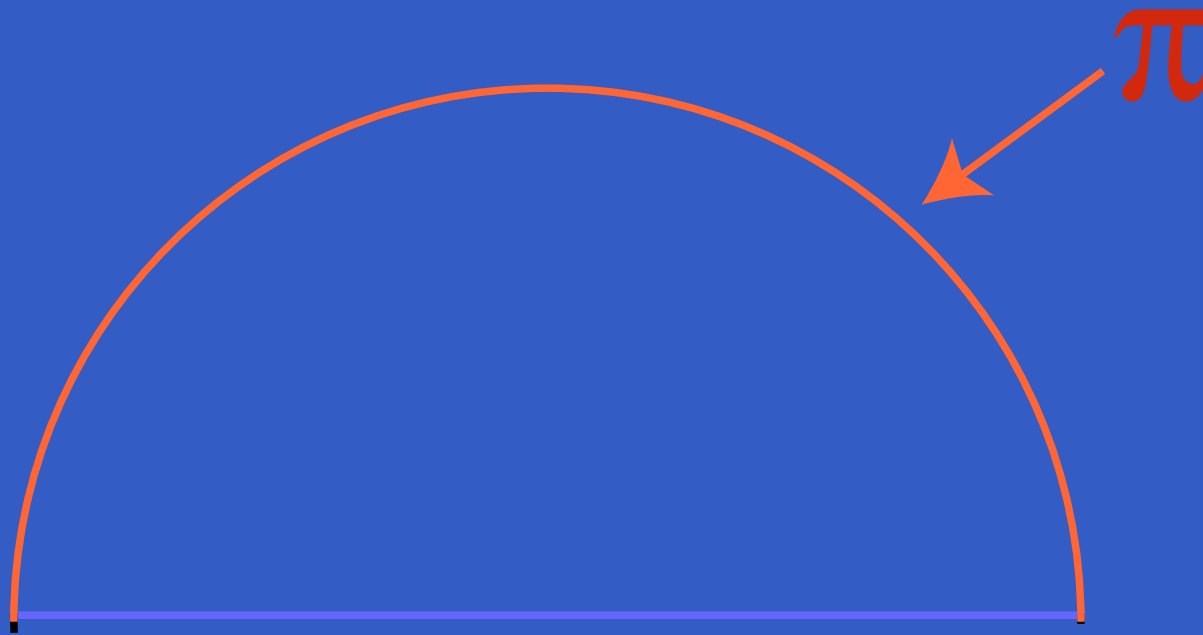
$$f(x_1, \dots, x_n) = \frac{x_1 + \dots + x_n}{n} - \sqrt[n]{x_1 \dots x_n}$$

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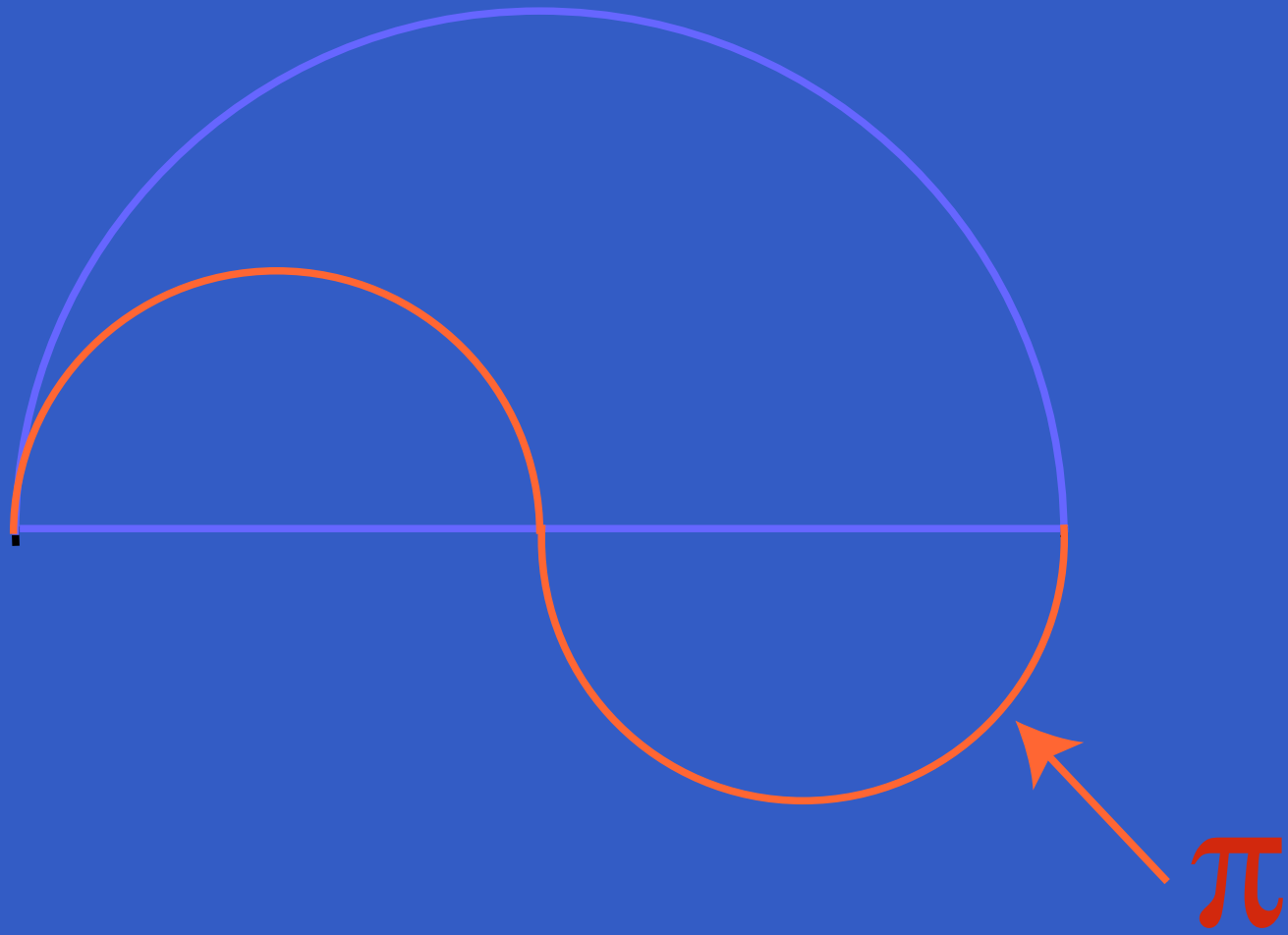
$$f(a_1, a_2, \dots, a_{n-1}, a_n) > f\left(\frac{a_1 + a_n}{2}, a_2, \dots, a_{n-1}, \frac{a_1 + a_n}{2}\right) \blacksquare$$

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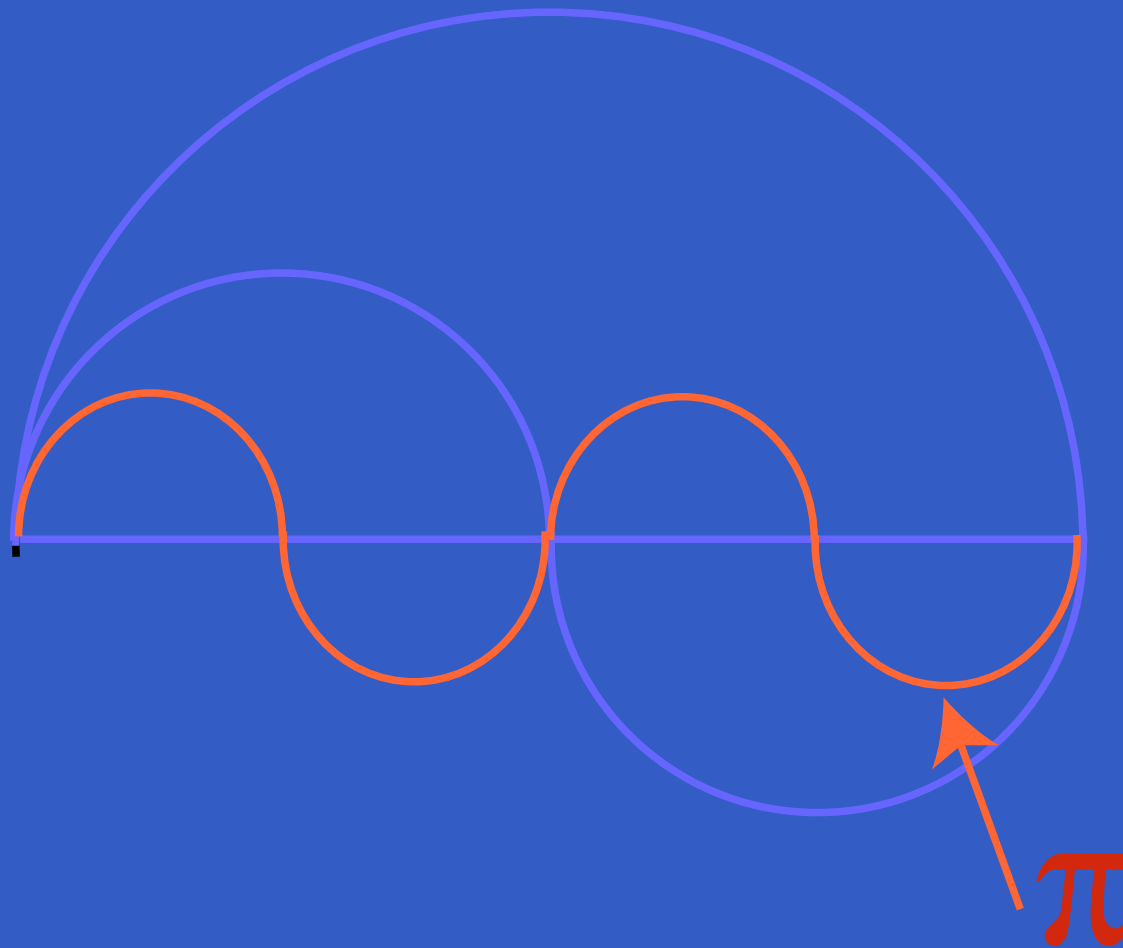
Calculus III



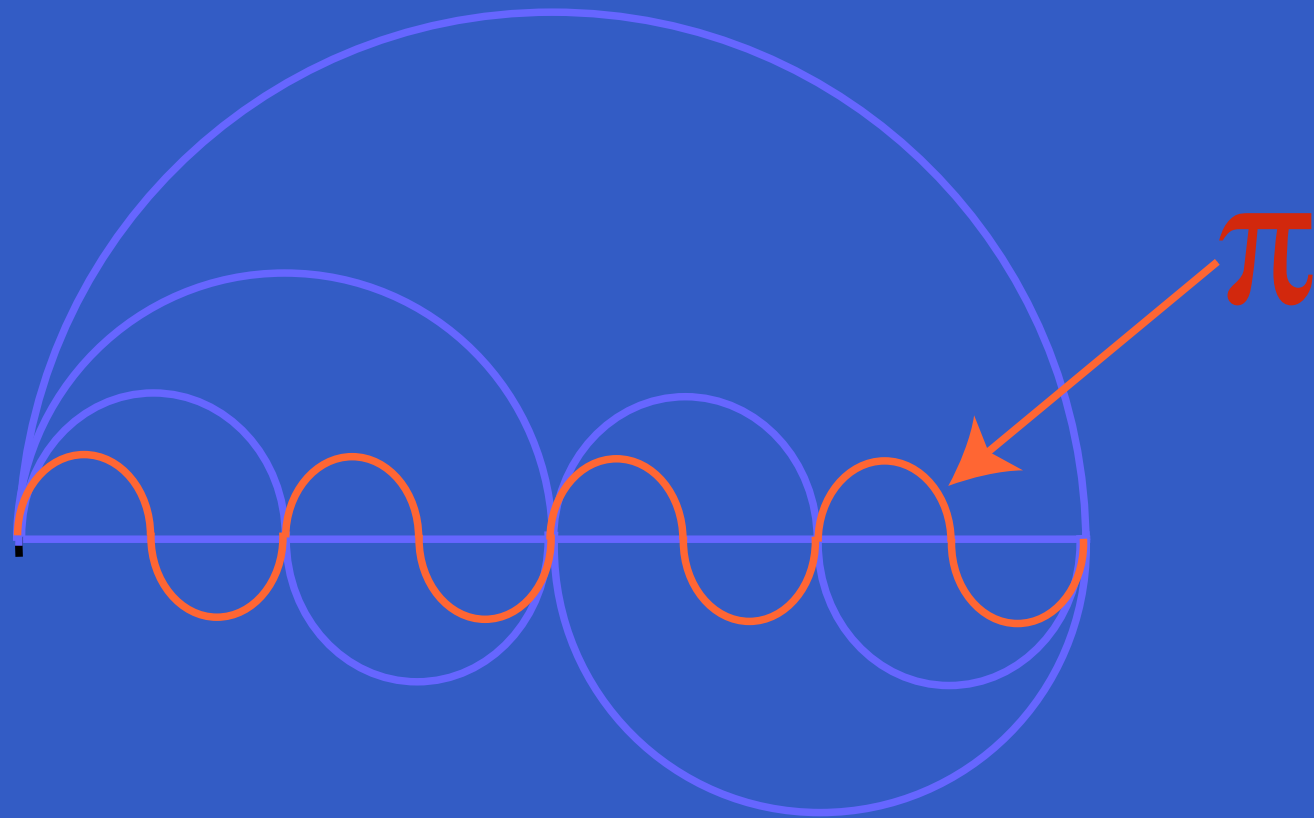
Calculus III



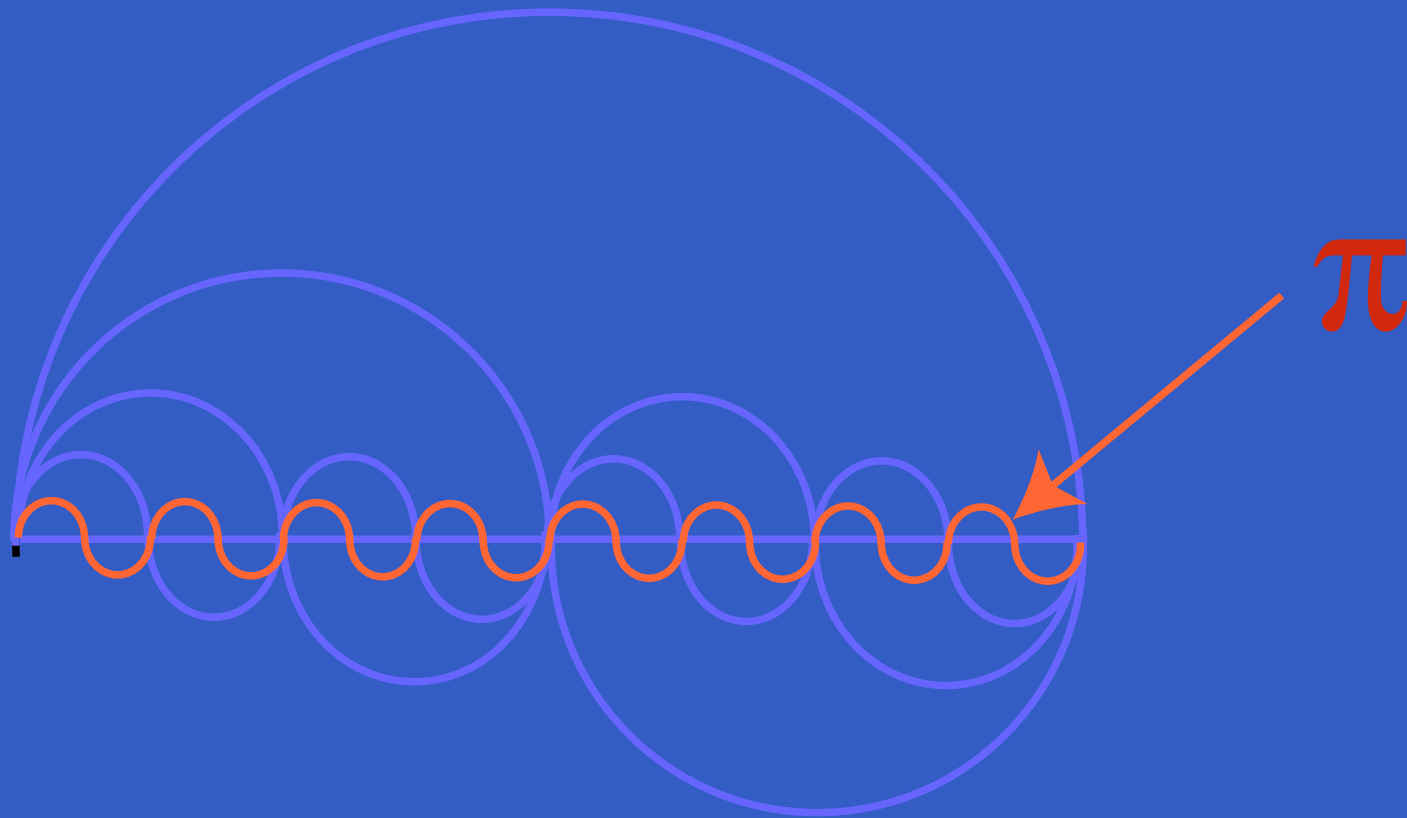
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